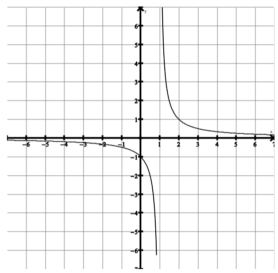
AP CALCULUS	LECTURE NOTES	MR. RECORD
Section Number: **3.4**	**Topics:** Concavity	**Day:** 1 of 2

Consider the following function:

$$f(x)=\frac{1}{x-1}$$



Note: $f(x)$ is concave downward on $(-\infty,1)$ and concave upward on $(1,\infty)$

Definition of Concavity

Let *f* be differentiable on an open interval *I*. The graph of *f* is **concave upward** on *I* if *f ′* is increasing on the interval and **concave downward** on *I* if *f ′* is decreasing on the interval.

Test for Concavity

Let *f* be a function whose second derivative exists on an open interval *I*.

1. If $f''(x)>0$ for all *x* in *I*, then the graph of *f* is concave upward in *I.*
2. If $f''(x)<0$ for all *x* in *I*, then the graph of *f* is concave downward in *I.*

Definition of a Point of Inflection

A point of inflection (p.o.i.) is an ordered pair where a graph changes concavity.

Points of Inflection Theorem

If $(c, f(c))$ is a point of inflection of the graph of *f*, then either $f''(x)=0$ or *f* is not differentiable at *x* = *c*.

Example 1: Determine the open intervals on which each graph is concave upward or downward and state any points of inflection.

a. $f(x)=\frac{6}{x^2+3}$

b. $f(x)=\frac{x^2+1}{x^2-4}$

c. $f(x)=x^4-4x^3$

AP CALCULUS	LECTURE NOTES	MR. RECORD
Section Number: **3.4**	**Topics:** The Second Derivative Test	**Day:** 2 of 2

The Second Derivative Test

Let *f* be a function such that $f'(x) = 0$ and the second derivative of *f* exists on an open interval containing *c*.

1. If $f''(x) > 0$, then $f(c)$ is a relative minimum.
2. If $f''(x) < 0$, then $f(c)$ is a relative maximum.

Example 2: Find the relative extrema for $f(x) = -3x^5 + 5x^3$ using the Second Derivative Test.

Example 3: Given the values below for *x*, $f'(x)$ and $f''(x)$, answer each of the following.

x	-3	-1	1	3	5
$f(x)$	-2	1	-1	-4	3
$f'(x)$	1	0	-1	0	2
$f''(x)$	-2	-1	0	2	3

a. Identify all *x*-values where *f* has a relative minimum. Justify.

b. Identify all *x*-values where *f* has a relative maximum. Justify.

c. Identify all *x*-values where *f* has a point of inflection. Justify.

d. What is the equation of the tangent to the curve *y* = *f*(*x*) at *x* = 5?